CS275 Discrete Mathematics

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Goal for labs

- Review contents
- Practice for homeworks/tests
- Answer questions
- Help you better understand the course & get the grade you aimed

Logic and Proof
Section 1.1 - 1.6

What is a **proposition**?

- A **proposition** is a declarative statement that is **True** or **False** but not both.
  - E.g., Tony is original from China.

Negation of a proposition (¬p)

- ¬p : it is not the case that p
  - E.g.,
    - p = true, ¬p = false
    - P = “Today is Wed.”, ¬p = “Today is NOT Wed.”

Conjunction of *p* and *q* (p∧q)

- The conjunction p∧q (p and q) is true if both p and q are true; otherwise it is false.
  - E.g.,
    - p = “Today is Wed.”, q = “Today is 01/01.”
    - p∧q = ?

Disjunction of *p* and *q* (pvq)

- The disjunction pvq (p or q) is false if both p and q are false; otherwise, it is true.
  - E.g.,
    - p = “Today is Wed.”, q = “Today is 01/01.”
    - pvq = ?

Conditional statement ($p \rightarrow q$)

- The conditional statement $p \rightarrow q$ (if p then q) is false when p is true and q is false; otherwise, it is true.
  - E.g., p = “If I have a keyboard”, q = “I can type”
    - If p is true, q is true, $p \rightarrow q$ is true
    - If p is false, q is false
      - “If I DON’T have a keyboard, I CAN’T type”
        - Could be! Thus, $p \rightarrow q$ = true
    - If p is false, q is true,
      - “If I don’t have a keyboard, I still type”, thus, $p \rightarrow q$ is true
    - If p is true, q is false,
      - “If I have a keyboard, I cannot type”
        - Why?! Thus, $p \rightarrow q$ = false

Conditional statement ($p \to q$)

- **$p \to q \equiv \neg p \lor q$**
  - E.g., p = “If I have a keyboard”, q = “I can type”
    - If p is true, q is true
      - $\neg p \lor q$: “if I don’t have a keyboard, I can type” TRUE
    - If p is false, q is false
      - $\neg p \lor q$: “if I have have a keyboard, I cannot type” FALSE
    - If p is false, q is true
      - $\neg p \lor q$: “if I don’t have a keyboard, I still can type” TRUE
    - If p is true, q is false
      - $\neg p \lor q$: “if I don’t have a keyboard, I cannot type” TRUE

Prove $p \to q \equiv \neg p \vee q$ (using truth table)

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Converse of $p \rightarrow q$ & Contraposition of $p \rightarrow q$

- Converse of $p \rightarrow q$ is $q \rightarrow p$
- The contraposition of $p \rightarrow q$ is $\neg q \rightarrow \neg p$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Biconditional statement (p↔q)

- A biconditional statement p↔q (if p and only if q) is true if both operands are true or both operands are false

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercise: Prove $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ using truth table

p	q	$p \leftrightarrow q$	$p \land q$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$(p \land q) \lor (\neg p \land \neg q)$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T